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Antihydrogen at sub-Kelvin temperatures

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We discuss the behavior of magnetically trapped antihydrogen ($\bar{H}$) at temperatures relevant for gravity and spectroscopy experiments (well below 1 K) and the possibilities of attaining these temperatures. Two possible options are considered. In the discussion of the first one, i.e. $\bar{H}$ as admixture in cold H gas, we develop the quantum-mechanical theory of $\bar{H}$–H (and also $\bar{p}$–H) elastic and rearrangement collisions at ultra-low (sub-Kelvin) energies, when s-wave scattering in the incoming channel dominates. The rate constant of rearrangement leading to $\bar{H}$ decay turns out to be large, which makes the possibilities for $\bar{H}$ collisional cooling in H gas and $\bar{H}$–H coexistence rather limited. As we show, the most promising is the other option, i.e. $\bar{H}$ atoms in the collisionless regime. For this regime the possibility of one-dimensional adiabatic cooling of $\bar{H}$ is demonstrated by using the example of the Ioffe trap. This phenomenon, interesting from the fundamental point of view, offers the opportunity to cool $\bar{H}$ below 1 mK.

1. Introduction

In recent successful experiments up to 10^5 low-energy antiprotons were accumulated and studied in an electromagnetic trap at low temperature (~ 4 K) [1]. These experiments open a realistic possibility to create and study the bound state of the antiproton with a positron, i.e. antihydrogen ($\bar{H}$). Being the simplest representative of neutral antimatter, $\bar{H}$ attracts great interest from several points of view. The creation of antihydrogen offers a unique opportunity to test CPT-invariance and the weak equivalence principle by gravity experiments with $\bar{H}$ [2]. Precise CPT checks can also be obtained from spectroscopic measurements of the 1S–2S transition frequency, Lamb shift and hyperfine transition frequency [3]. Especially interesting is also atom–antiatom interaction in a dilute gaseous mixture of antihydrogen and hydrogen (H), first of all in relation to the fundamental problem of matter–antimatter coexistence.

In this paper we restrict ourselves to the most interesting option in which antihydrogen will be produced with sufficiently low kinetic energy to enable trapping in a neutral-atom trap and subsequent cooling to temperatures below approximately 1 mK, where the conditions for gravity experiments and precision spectroscopy are optimal from an experimental point of view [4,5]. Trapping is

essential for accumulation of the produced antiatoms and to avoid annihilation in collisions of $\bar{H}$ with surfaces. As was first demonstrated for the neutron, surface free confinement is possible in a local magnetic field minimum for neutral particles with magnetic moments antiparallel to the magnetic field (low field seekers) [6].

For atomic hydrogen (H) trapping was demonstrated in recent experiments with the metastable spin-up polarized gaseous phase, H↑ (electron magnetic moment antiparallel to the field), at sub-Kelvin temperatures and densities n_H in the range 10^{11}–10^{14} cm^{-3} [7–10]. The stability of H↑ is limited by spin relaxation in pair collisions due to the magnetic dipolar interaction between the atoms. This results in the appearance of H↓ atoms which are repelled from the trap. The rate constant of this process $\alpha_{rel} \approx 10^{-15}$cm^3/s, and the corresponding characteristic lifetime of the system, $\tau_H = (\alpha_{rel} n_H)^{-1}$, ranges from 10^4 to 10 s for the above mentioned respective range of densities. In practice, the magnetic well depth for H↑ cannot exceed several Kelvin, which implies the condition $T \ll 1$ K for the temperature of H↑ gas to keep the atoms trapped.

The lowest temperatures that can be anticipated for $\bar{H}$ depend strongly on the selected method of cooling. Laser cooling is expected to be of limited use for $\bar{H}$ as photon recoil will limit the minimum attainable temperature to ~ 1 mK. Much lower temperatures can be reached with evaporative cooling as has been observed in experiments with H [10]. In this method atoms with an energy that exceeds a magnetic potential barrier (Δ) may escape from the trap leaving the remaining gas at a reduced temperature. Since such atoms are mainly produced in elastic pair collisions, the rate of evaporative cooling is proportional to the rate of elastic collisions $\tau_c^{-1} \approx n_H \sigma \langle v \rangle$ ($\sigma \approx 10^{-15}$ cm^2 is the elastic scattering cross section, $\langle v \rangle$ is the thermal velocity), which corresponds to a collisional time of tens of seconds for $n_H \approx 10^{11}$cm^{-3} and $T \approx 1$ mK. It is worth mentioning that the characteristic cooling rate $\tau_{ev}^{-1} \sim \tau_c^{-1} \exp(-\Delta/T)$ has to be chosen much slower than the elastic collision rate but much faster than the magnetic relaxation rate (magnetic relaxation gives rise to internal heating of the gas). The theoretical limit for evaporative cooling is reached once this requirement can no longer be satisfied and turns out to be below approximately 1 μK. At present, the lowest temperature realized by evaporative cooling of H is $T \approx 100$ μK [10].

Since the spin states of $\bar{H}$ in a magnetic field are identical to those of H (only magnetic moments of particles should be inverted), one may expect that in the case of surface-free confinement the properties of $\bar{H}$↓ will be similar to that of H↑. However, there is a principle difference resulting from the much lower antihydrogen densities n that can be attained at present facilities. Even under most favorable conditions n will be many orders of magnitude smaller than typical H↑ densities ($n_H \approx 10^{11} - 10^{14}$ cm^{-3}). This makes evaporative cooling into an inapplicable technique, since $\tau_{ev} \approx n^{-1}$ should be anomalously large unless very small samples can be realized. Thus, recalling the limitations of optical cooling, it definitely requires serious consideration of other mechanics to cool $\bar{H}$ below 1 mK.

In this perspective, there are two possible options for studying $\bar{H}$. The first one is to trap $\bar{H}\downarrow$ in the collisionless regime, i.e. at extremely low densities, with the possibility of adiabatic cooling in mind. The second option is to introduce $\bar{H}\downarrow$ as a small admixture into magnetically trapped $H\uparrow$ gas (or another trapped gas) and to study the combined system. Aside from the prospect of collisional cooling of $\bar{H}$, it provides us with an interesting view on the fundamental aspects of the atom–antiatom interaction as it manifests itself in collisional phenomena.

2. $\bar{H}$–H and $\bar{p}$–H collisions at sub-Kelvin energies

In this section we shall discuss collisional physics for $\bar{H}$ in H gas at ultra-low (sub-Kelvin) energies, which involves both elastic and inelastic binary collisions. In the limit of such energies s-wave scattering dominates (see below), and the relative motion of colliding particles should be treated quantum-mechanically. We shall do the same for $\bar{p}$–H collisions, since these were actively studied at higher energies corresponding to classical motion of colliding particles in the region of interaction. This activity originates in the fact that μ^-–H collisions, similar to those of $\bar{p}$–H, are important for the problem of muon catalyzed fusion.

The collisional physics aspects are, first of all, determined by the interaction potential $U(R)$ (R is the internuclear distance) between the colliding particles. In the case of $\bar{p}$–H the interaction is fully attractive (see, e.g., ref. [11]). The function $U(R)<0$ for $R>a_0$ rises monotonously with increasing internuclear separation R and acquires at large R the asymptotic form $U(R) = -\frac{1}{2}\alpha e^2/R^4$ ($\alpha = \frac{9}{2}a_0^3$ is the polarizability of the hydrogen atom, a_0 is the Bohr radius, e is the electron charge). At distances $R<a_0$ the adiabatic approximation breaks down – there are no bound states any more for the electron in the field of the $\bar{p}$–p dipole. Actually, at these short distances the potential curve $U(R)$ joins the ionization curve $U_c(R) = -e^2/R + \frac{1}{2}e^2/a_0$ (which corresponds to the $\bar{p}$–p interaction), and a rearrangement reaction (adiabatic ionization) may occur. The antiproton picks up a proton and forms an excited bound state of protonium, and the electron is ejected,

$$\bar{p} + H \rightarrow Pn^* + e. \tag{1}$$

The Pn^* rapidly decays through annihilation.

The interaction potential $U(R)$ for $\bar{H}$–H was calculated by Kolos et al. [12]. The potential curve is of the same shape as in the case of $\bar{p}$–H, the long-range tail being $U(R) = -C_6/R^6$ ($C_6 = 6.5e^2a_0^5$ is the van der Waals constant). The adiabatic approximation also breaks down at $R<a_0$, and the rearrangement processes occurring at these distances are the following (see ref. [11]):

$$\bar{H} + H \rightarrow Pn^* + Ps,$$

$$\bar{H} + H \rightarrow Pn^* + e^+ + e^- \tag{2}$$

with subsequent annihilation decay of protonium and positronium. The shape of $U(R)$, typical for $\bar{p}$–H and $\bar{H}$–H interactions, is presented in fig. 1.

It is very essential that there is no potential barrier in the $\bar{H}$–H interaction. Various speculations on this subject (see ref. [13] and references therein) were finished after the publication of Kolos et al. [12]. The absence of such a barrier means that in the case of s-wave scattering nothing prevents the particles from approaching each other to the short internuclear distances at which the rearrangement processes (2) typically occur. Thus, at ultra-low collision energies, the rearrangement probability should be rather large and therefore easily observable. On the other hand this feature also limits the coexistence lifetime, which makes $\bar{H}$–H mixtures less suited for applications such as collisional cooling.

We emphasize that the absence of a potential barrier in the $\bar{H}$–H case does not mean that such a barrier is absent for the interaction of $\bar{H}$ with any other collisional partner. Therefore, the search for a collisional partner with a barrier in the interaction potential remains very important in relation to atom–antiatom coexistence and $\bar{H}$ collisional cooling.

In the subsequent discussion of inelastic processes we shall confine ourselves to rearrangement collisions only. Spin relaxation in $\bar{H}$–H collisions, due to the magnetic dipole interaction, should proceed at approximately the same rate as in the case of H↑–H↑ ($\alpha_{rel} \approx 10^{-15}$ cm^3/s). With this number, it can be easily estimated that the spin relaxation is much slower than rearrangement and does not play an essential role.

For $\bar{p}$–H and $\bar{H}$–H interaction potentials the integral $\int U(R)\, d^3R$ is convergent at large distances R, which allows us to introduce the effective radius of interaction, R_e, for s-wave scattering in the $k \to 0$ limit, where k is the wave vector for the relative motion. In the case of a power-law potential $U(R)$ the radius R_e is

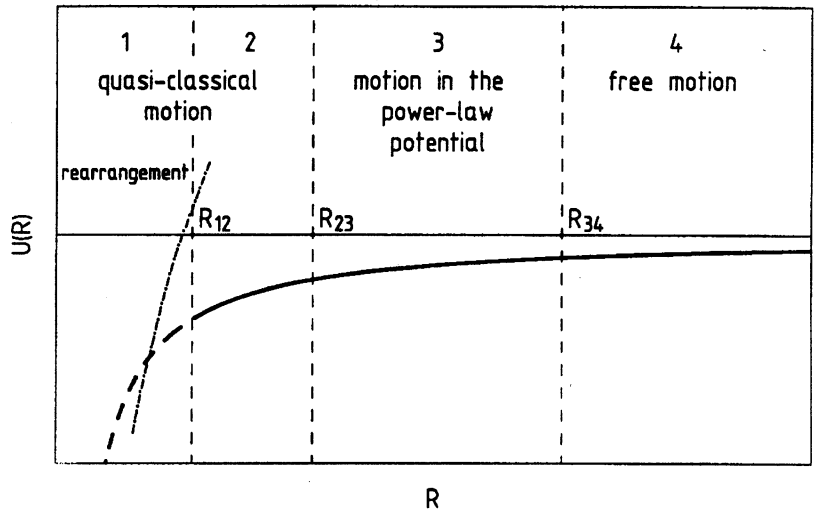

Fig. 1. The potential $U(R)$ typical for $\bar{p}$–H and $\bar{H}$–H.

determined by the condition that the kinetic energy term in the Schrödinger equation equals the potential energy term,

$$\frac{\hbar^2}{2MR_e^2} = U(R_e) \tag{3}$$

(M is the reduced mass of the colliding particles) and the main contribution to the phase shift δ_0 comes from distances $R \lesssim R_e$. For $\bar{p}$–H and $\bar{H}$–H collisions we have

$$\begin{aligned} R_e &= R_{e\bar{p}} = (\alpha e^2 M/\hbar^2)^{1/2} \approx 65a_0, \quad \bar{p}\text{–H}; \\ &= R_{e\bar{H}} = (2C_6 M/\hbar^2)^{1/4} \approx 10a_0, \quad \bar{H}\text{–H}. \end{aligned} \tag{4}$$

The limit of ultra-low energies in which we are interested (s-wave scattering dominates), is determined by the condition

$$kR_e \ll 1 \tag{5}$$

and corresponds to collisional energies

$$\begin{aligned} E &\ll 40\,\text{mK}, \quad \bar{p}\text{–H}; \\ &\ll 1.5\,\text{K}, \quad \bar{H}\text{–H}. \end{aligned}$$

Before discussing limit (5) let us give a brief outline of the opposite limit, $kR_e \gg 1$, where the relative motion of colliding particles can be considered as classical. This limit was rather well investigated previously, especially at electron-volt energies, where $\bar{p}$–H and also $\bar{H}$–H collisions are dominated by the rearrangement processes (1) and (2) (see, e.g. ref. [11]). The simplest approach is based on the assumption that rearrangement occurs with probability equal to unity if the particles approach each other to short internuclear distances ($R \lesssim a_0$). The, the rearrangement cross section coincides with the cross section of capture in the effective potential $U_{\text{eff}}(R) \equiv U(R) + E(\rho/R)^2$ (ρ is the impact parameter of the collision), i.e. any collision with energy exceeding the height of the centrifugal barrier leads to rearrangement. Below energies of approximately 1 eV the capture cross section σ_c is determined by the long-range tail of the potential $U(R)$ (polarization tail for $\bar{p}$–H and van der Waals tail for $\bar{H}$–H). The rate constant of rearrangement $\alpha_{in} = \alpha_c = \langle \sigma_c v \rangle$ for the $\bar{p}$–H case becomes

$$\alpha_{in} = 2\pi\hbar R_{e\bar{p}}/M \approx 10^{-9}\text{cm}^3/\text{s}, \quad kR_{e\bar{p}} \gg 1. \tag{7}$$

For the case of $\bar{H}$–H

$$\begin{aligned} &\alpha_{in} = (3\pi\hbar R_{e\bar{H}}/M)(kR_{e\bar{H}}/4)^{1/3} \approx 1.5 \times 10^{-10}(E/1\,\text{K})^{1/6}\ \text{cm}^3/\text{s}, \\ &kR_{e\bar{H}} \gg 1 \end{aligned} \tag{8}$$

(the numerical coefficients given here are slightly different from those in the original paper [11] where a more detailed shape for the long-range part of $U(R)$ was taken into account).

These results can be improved by introducing the ionization width $\Gamma(R)$, i.e. the probability of ionization per unit time at fixed internuclear separation (see, e.g., refs. [14,15]). This quantity was calculated for the case of $\bar{p}$–H (also for $\bar{p}$–He) in the Born–Oppenheimer approximation by using the diabatic curve representation for the initial state [14,15]. Such a curve crosses the ionization curves of some protonium states formed in the rearrangement process. Since there are many such states, quantum interference effects should be essentially smeared out, and for the ionization probability per capture collision we obtain

$$W = 1 - \exp\left(-\int_0^\infty [\Gamma(R)/v(R)]\mathrm{d}R\right), \tag{9}$$

where $v(R)$ is the classical velocity at internuclear distance R. This expression also assumes that $v(R)$ is independent of the impact parameter ρ, since in the region of rearrangement the energy of interaction between particles is much larger than the centrifugal energy $E(\rho/R)^2$ at ρ leading to capture. With eq. (6) taken into account, the rearrangement rate constant is

$$\alpha_{\mathrm{in}} \approx \alpha_{\mathrm{c}} W. \tag{10}$$

The value of the classical ionization probability W for $\bar{p}$–H, calculated for low collision energy with the $\Gamma(R)$-data of Cohen et al. [14] is close to 0.8. The same number was obtained by Morgan (see ref. [16]). For the $\bar{H}$–H case W was not calculated as data for $\Gamma(R)$ are not available in the literature. Nevertheless, we expect that in this case W will be rather close to that of $\bar{p}$–H.

The classical ionization probability W is a very important quantity for the region of ultra-low energies satisfying the condition (5). As will be shown below, in this region W also appears in the relations for the elastic and inelastic cross sections. The chief idea, allowing us to get a result under condition (5), is based on the fact that even at extremely low collisional energies E the relative radial motion of particles is quasiclassical at small enough internuclear distances. This directly follows from the shape of the potential $U(R)$. For $E \to 0$ the WKB criterion (see, e.g., ref. [17]), determining the region of R where the quasiclassical approximation is valid, gives

$$R \ll R_{\mathrm{e}\bar{\mathrm{p}}}, \qquad \bar{\mathrm{p}}\text{–H}; \tag{11}$$

$$R^2 \ll \tfrac{1}{2} R^2_{\mathrm{e}\bar{\mathrm{H}}}, \quad \bar{\mathrm{H}}\text{–H}. \tag{12}$$

At these distances (but outside the rearrangement region) the expression for the radial wave function is different from the ordinary quasiclassical expression only by a normalization coefficient. Since $R_{\mathrm{e}\bar{\mathrm{p}}}$ and $R_{\mathrm{e}\bar{\mathrm{H}}}$ are much larger than the characteristic distances at which rearrangement occurs, it becomes clear why the classical ionization probability is present in the expression for the inelastic (σ_{in}) and elastic (σ_{el}) cross sections. However, the dependence of σ_{el} and σ_{in} on W is different from that in the case of $kR_{\mathrm{e}} \gg 1$ as a consequence of the quantum-mechanical character of the radial motion at $R \sim R_{\mathrm{e}}$. Under the condition (5) with inelastic

processes (rearrangement) taken into account, the scattering length a in the expression for the phase shift of s-scattering, $\delta_0 = -ka$, acquires an imaginary part (see, e.g., ref. [17]),

$$a = a_r - ia_{im}, \quad a_{im} > 0. \tag{13}$$

The expressions for the rate constants of elastic scattering and rearrangement become (see, e.g., ref. [17])

$$\alpha_{el} = \sigma_{el} v = 4\pi\hbar k(a_r^2 + a_{im}^2)/M, \tag{14}$$

$$\alpha_{in} = \sigma_{in} v = 4\pi\hbar a_{im}/M. \tag{15}$$

Thus, in accordance with the general character of the energy dependence, $\alpha_{el} \sim k \sim \sqrt{T}$ and $\alpha_{in} =$ constant, this implies the existence of a limiting temperature below which $\alpha_{in} \gtrsim \alpha_{el}$, the coexistence of $\bar{H}$ and H in thermal equilibrium is impossible and collisional cooling of $\bar{H}$ is not effective. One of our goals is to determine α_{el} and α_{in} and clarify whether there are collisional energies for which $\alpha_{el} > \alpha_{in}$.

Let us consider s-wave scattering under the condition (5) and divide the full range of internuclear distances into four regions, as shown in fig. 1. In region 1 ($R < R_{12} \approx a_0$) the rearrangement processes occur. The dashed line represents the diabatic potential curve for the initial state, and the dash-dotted line corresponds to the ionization curve of one of the final states. The motion along these curves can be treated quasiclassically. In region 2 there is no rearrangement, but the radial motion is also quasiclassical. The characteristic distance R_{23} satisfies criterion (11) or (12), but is essentially larger than $R_{12} \sim a_0$. Region 3 corresponds to motion in the power-law potential and in region 4 we have free particle motion.

It is essential that region 2 overlaps with region 3. The characteristic distance R_{23}, satisfying condition (11) or (12), can be chosen such that the power-law dependence of the potential $U(R)$ already holds. This approach works well for $\bar{p}$–H and slightly worse in the $\bar{H}$–H case. Naturally, at large enough R region 3 goes over into region 4. The characteristic distance R_{34} should be much larger than R_e, and the condition (5) allows us to choose it such that $kR_{34} \ll 1$.

The overlap mentioned above enables a matching procedure in which the wave functions are properly joined. Since even R_{34} satisfies the condition $kR_{34} \ll 1$, all expressions for the wave functions used in this procedure are solutions of the Schrödinger equation with $k = 0$.

In region 2, where the motion is quasiclassical, we first represent the wave function as a superposition of an incoming and outgoing wave,

$$\psi_{inc} \sim \exp\left(i \int_{R_{12}}^{R} k_0(R')\mathrm{d}R'\right), \tag{16}$$

$$\psi_{out} \sim \exp\left(-i \int_{R_{12}}^{R} k_0(R')\mathrm{d}R' - A/2 - 2i\tilde{\Phi}\right), \quad A > 0, \tag{17}$$

where

$$k_0(R) = \sqrt{2M|U(R)/\hbar^2}. \tag{18}$$

Due to rearrangement processes in region 1 the outgoing flux in region 2 is smaller than the incoming one, which is described by the factor $\exp(-A/2)$ in eq. (17). In the usual representation for the radial wave function we have

$$\psi_2 = \frac{B}{R[k_0(R)]^{1/2}} \sin\left(\int_{R_{12}}^{R} k_0(R')\,\mathrm{d}R' - \mathrm{i}A/4 + \tilde{\Phi}\right). \tag{19}$$

The real part of the phase shift $\tilde{\Phi}$ is determined by the behavior of the system at short internuclear distances and can be obtained with sufficient accuracy only by a precise numerical calculation (solution of three-body problem for $\bar{\mathrm{p}}$–H and four-body problem for $\bar{\mathrm{H}}$–H).

Before performing the matching procedure for the wave function let us clarify the physical meaning of the quantity A and, with this purpose in mind turn to the "high-energy case" ($kR_e \gg 1$). In this case outside the rearrangement region the wave function for ℓ-wave scattering is determined by the ordinary quasiclassical expression which can be written in the form

$$\psi_\ell = \frac{b_\ell(E)}{R(k_{E\ell}(R))^{1/2}} \sin\left(\int_{R_{12}}^{R} k_{E\ell}(R')\,\mathrm{d}R' - \mathrm{i}A_\ell(E)/4 + \tilde{\Phi}_\ell(E)\right) \tag{20}$$

with

$$k_{E\ell}(R) = \sqrt{2M[E - U(R) - \hbar^2\ell(\ell+1)/2MR^2]/\hbar^2}. \tag{21}$$

As mentioned above, for $\ell \to 0$ and $E \to 0$ the rhs of eq. (20) should be different from the rhs of eq. (19) only by a normalization coefficient and, hence, $A_0(0) = A$. Relation (20) is valid for $\ell \leqslant \ell_{\max}(E)$, where $\ell_{\max}(E)$ is the maximum value of the orbital moment of the collision at which the collision energy exceeds the height of the centrifugal barrier. For $\ell > \ell_{\max}(E)$ the particles are reflected by the barrier and do not approach each other to the internuclear distances corresponding to regions 1 and 2 (i.e. rearrangement does not occur). The quantity $\ell_{\max}(E)$ is related to the capture cross section through

$$\sigma_c = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\max}} (2\ell + 1). \tag{22}$$

It is important that for $\ell \leqslant \ell_{\max}(E)$ and not *very* high collisional energy ($E \lesssim 1\mathrm{eV}$) the parameters $A_\ell(E)$ are practically independent of ℓ and E, since under such conditions the interaction energy in the rearrangement region $|U| > |U(R_{12})| \gg (E, \hbar^2\ell(\ell+1)/2MR^2)$. Thus we have

$$A_\ell(E) \approx A_0(0) = A, \tag{23}$$

where A is the parameter appearing in eq. (19). The diagonal element of the scattering S-matrix for $\ell \leqslant \ell_{\max}(E)$ is

$$S_\ell(E) = \exp\left(-A_\ell(E)/2 + 2i\tilde{\Phi}_\ell(E) + 2i\int_{R_{12}}^{\infty}[k_{E\ell}(R') - k]\,dR'\right). \tag{24}$$

For $\ell > \ell_{\max}$ we have the same expression, but with $A_\ell(E) = 0$ and a slightly different imaginary part in the exponent. Then, using eqs. (22) and (23) and the general expression for the total rate constant of inelastic scattering (see ref. [17]) for the rearrangement rate constant we obtain

$$\alpha_{\text{in}} = \frac{\pi}{k^2} v \sum_{\ell=0}^{\ell_{\max}} (2\ell+1)(1 - |S_\ell(E)|^2) = \alpha_c(E)[1 - \exp(-A)], \tag{25}$$

where $\alpha_c(E) = \langle \sigma_c v \rangle$ is the capture rate constant. Comparing eqs. (25) and (10) we immediately see that $1 - \exp(-A)$ is nothing else than the classical ionization probability per capture collision,

$$W = 1 - \exp(-A). \tag{26}$$

If the representation of the diabatic potential curve is used for the $\bar{p}$–H and $\bar{H}$–H interaction in the rearrangement region, then

$$A = \int_0^{\infty} [\Gamma(R)/v(R)]dR \tag{27}$$

(the main contribution comes from distances in region 1). However, we should especially emphasize that eq. (26) and our interpretation of the parameter A are of general character and do not depend on the approach for the calculation of A. We only used the fact that the parameters $A_\ell(E)$ are independent of orbital moment and energy. This general character is important, since various methods (not only the diabatic curve representation) can be applied for finding A and W (cf. refs. [14,16]).

Let us now turn to our ultra-low energy case ($kR_e \ll 1$). At distances close to R_{23} the function ψ_2 (19) should change over into the function ψ_3 which is the exact solution of the Schrödinger equation in the power-law potential $U(R)$. On the other hand, for R close to $R_{34} \gg R_e$ the function ψ_3 goes over into the wave function of free motion at $k = 0$,

$$\psi_4 = 1 - a/R. \tag{28}$$

Thus, the wave function matching procedure allows us to express the coefficient B and scattering length a in terms of A (or W) and $\tilde{\Phi}$.

Let us first discuss the case of $\bar{p}$–H, corresponding to the long-range tail of the potential $U(R) \approx -\frac{1}{2}\alpha e^2/R^4$. The solution of the Schrödinger equation for this potential, i.e. the wave function in region 3, which assumes its asymptotic form (28) for $R \gg R_{e\bar{p}}$, is

$$\psi_3 = \cos(R_{e\bar{p}}/R) - (a/R_{e\bar{p}})\sin(R_{e\bar{p}}/R). \tag{29}$$

For $R \ll R_{e\bar{p}}$ this expression should take the form (19). Representing the argument of the sin in (19) by $\Phi - iA/4 - R_{e\bar{p}}/R$ with $\Phi = \tilde{\Phi} + \int_{R_{12}}^{\infty} k_0(R)\,dR$, we obtain the real and imaginary part of the scattering length,

$$a_r = R_{e\bar{p}} \frac{\sin\Phi\cos\Phi}{\sinh^2(A/4) + \sin^2\Phi}, \tag{30}$$

$$a_{im} = R_{e\bar{p}} \frac{\sinh(A/4)\cosh(A/4)}{\sinh^2(A/4) + \sin^2\Phi}. \tag{31}$$

With allowance for these results, eqs. (14) and (15) lead to the following relations for the rate constants of elastic scattering and rearrangement:

$$\alpha_{el} = \frac{4\pi\hbar R_{e\bar{p}}}{M}(kR_{e\bar{p}}) \frac{\sin^2\Phi\cos^2\Phi + \cosh^2(A/4)\sinh^2(A/4)}{[\sinh^2(A/4) + \sin^2\Phi]^2}, \tag{32}$$

$$\alpha_{in} = \frac{4\pi\hbar R_{e\bar{p}}}{M} \frac{\sinh(A/4)\cosh(A/4)}{\sinh^2(A/4) + \sin^2\Phi}. \tag{33}$$

The quantum character of the radial motion at $R \sim R_{e\bar{p}}$ manifests itself in two ways. First of all, the dependence of the rate constants on the ionization probability is different from that in the classical case (cf. eq. (10)). Then, eqs. (32) and (33) contain the factors $\sin\Phi$ and $\cos\Phi$. It is interesting to note that for $W = 1$ ($A \to \infty$) the rearrangement rate constant $\alpha_{in} = 4\pi\hbar R_{e\bar{p}}/M$, i.e. twice as large as in the classical case.

There is an uncertainty in our final results due to the presence of the unknown factors $\sin\Phi$ and $\cos\Phi$ in eqs. (32) and (33). However, the minimum possible value of the ratio α_{in}/α_{el} is

$$\left(\frac{\alpha_{in}}{\alpha_{el}}\right)_{min} = \frac{1}{kR_{e\bar{p}}}\tanh(A/4). \tag{34}$$

With the value $W \approx 0.8$ ($A \approx 1.6$) mentioned above for the $\bar{p}$–H case, we obtain

$$\left(\frac{\alpha_{in}}{\alpha_{el}}\right)_{min} \approx \frac{0.3}{kR_{e\bar{p}}}. \tag{35}$$

For this value of W the characteristic values of the scattering parameters are the following. The elastic cross section $\sigma_{el} \sim 4\pi R_{e\bar{p}}^2 \sim 10^4\pi a_0^2$, which corresponds to a rate constant $\alpha_{el} \sim 3 \times 10^{-9}(T/40\,\mathrm{mK})^{1/2}\,\mathrm{cm}^3/\mathrm{s}$. The rate constant for rearrangement $\alpha_{in} \sim 10^{-9}\,\mathrm{cm}^3/\mathrm{s}$, i.e. rather close to its value in the classical case.

In the case of $\bar{H}$–H the long-range potential tail $U(R) \sim -C_6/R^6$. The solution of the Schrödinger equation for this potential, which reaches its asymptotic form (28) for $R \gg R_{e\bar{H}}$, is

$$\psi_3 = \Gamma(\tfrac{3}{4})\left(\frac{R_{e\bar{H}}}{2R}\right)^{1/2}\left[\mathbf{J}_{(-1/4)}\left(\frac{R_e\bar{H}^2}{2R^2}\right) - \frac{2a\Gamma(\frac{5}{4})}{R_{e\bar{H}}\Gamma(\frac{3}{4})}\mathbf{J}_{1/4}\left(\frac{R^2_{e\bar{H}}}{2R^2}\right)\right], \tag{36}$$

where $\mathbf{J}$ is the Bessel function. For distances R satisfying the condition (12) this function should go over into the quasiclassical expression (19). Representing the argument of the sin in (19) as $\pi/8 - iA/4 + \Phi - \frac{1}{2}R^2_{e\bar{H}}/R^2$ with $\Phi = \tilde{\Phi} + \hbar^{-1}\int_{R_{12}}^{\infty} k_0(R)\,dR - \pi/8$, we obtain real and imaginary parts of the scattering length a. The relations for the rate constants prove to be the following:

$$\alpha_{el} = \frac{4\pi\hbar R^*_{e\bar{H}}}{M}(kR^*_{e\bar{H}}) \times \frac{[\sinh^2(A/4) + \cos\Phi(\cos\Phi + \sin\Phi)]^2 + \sinh^2(A/4)\cosh^2(A/4)}{[2\sinh^2(A/4) + (\cos\Phi + \sin\Phi)^2]^2}, \tag{37}$$

$$\alpha_{in} = \frac{4\pi\hbar R^*_{e\bar{H}}}{M}\frac{\sinh(A/4)\cosh(A/4)}{[2\sinh^2(A/4) + (\cos\Phi + \sin\Phi)^2]}, \tag{38}$$

where $R^*_{e\bar{H}} = 2^{-1/2}\Gamma(\frac{3}{4})/\Gamma(\frac{5}{4})R_{e\bar{H}} = 0.96R_{e\bar{H}}$. Although these relations are slightly different from those in the case of $\bar{p}$–H, their qualitative interpretation is the same. For the minimum possible value of the ratio α_{in}/α_{el} we obtain the same expression as in the $\bar{p}$–H case, i.e. eq. (34) but with $R^*_{e\bar{H}}$ replacing $R_{e\bar{p}}$.

As mentioned above, for $\bar{H}$–H the value of W (or A) is not available in the literature. However, in view of strong similarities it should be rather close to that in the case of $\bar{p}$–H. Thus, $(\alpha_{in}/\alpha_{el})_{min}$ will be close to that from eq. (35). The characteristic values of scattering parameters are

$$\sigma_{el} \sim 200\pi a_0^2,$$

$$\alpha_{el} \sim 10^{-10}(T/1\mathrm{K})^{1/2}\,\mathrm{cm}^3/\mathrm{s},$$

$$\alpha_{in} \sim 0.3 \times 10^{-10}\,\mathrm{cm}^3/\mathrm{s}.$$

The results obtained show that the possibility for collisional cooling of $\bar{H}$ in H↑ gas and the possibility for $\bar{H}$–H coexistence are rather limited. These possibilities would be much more promising if the classical ionization probability W were small, since the possible minimum value of the ratio $(\alpha_{in}/\alpha_{el})$ decreases substantially with decreasing W and the search for a collisional partner with small W, that would enable long term coexistence, remains important.

3. Collisional cooling of $\bar{H}$

In this section we analyze the consequences of the results obtained above for experiments of $\bar{H}$–H or $\bar{p}$–H gaseous mixtures as they conceivably could be

performed in magnetic traps. It is interesting to establish to what temperatures $\bar{H}$ can be cooled by mixing it with H, in particular taking advantage of the evaporative cooling techniques that have been developed for magnetically trapped hydrogen.

The most straightforward approach, evaporative cooling of the $\bar{H}$–H mixture, seems to be fundamentally impossible. The characteristic value of the cross section for $\bar{H}$–H elastic scattering σ_{el} (see section 2) is much larger than that for H–H collisions, $\sigma_{el} \approx 30\sigma$. This implies that small quantities of $\bar{H}$ in H-gas will equilibrate with the gas on a time scale which is short in comparison to the characteristic time for evaporative cooling of H. Unfortunately, the rate constant for inelastic scattering α_{in} cannot be much smaller than α_{el}. It follows from eq. (35), that under best conditions, i.e. for $kR_e \approx 1$ ($T \approx 1.5$ K), we have $\alpha_{in} \approx 0.3\alpha_{el}$. This means that $\bar{H}$ is expected to be lost due to rearrangement collisions with H before the H temperature can be appreciably changed by evaporative cooling.

The only realistic option for cooling $\bar{H}$ with H- gas is therefore to first cool H and then to supply $\bar{H}$. To reveal the essence of this approach assume the H gas to have a sufficiently low temperature that per $\bar{H}$–H collision the (average) reduction of the $\bar{H}$ kinetic energy is simply a factor 2. Then, the kinetic energy loss rate per $\bar{H}$ will be

$$\dot{E} = -n_H v \sigma_{el} E/2, \tag{39}$$

where v is the $\bar{H}$–H relative velocity (which simply equals the $\bar{H}$ velocity as the H velocity was assumed to be negligible and, hence, $v = (E/M)^{1/2}$). The particle loss rate is

$$\dot{n} = -\alpha_{in} n_H n, \tag{40}$$

which gives

$$n = n_0 \exp\left(-\frac{t}{\alpha_{in} n_H}\right), \tag{41}$$

where n_0 is the initial density of $\bar{H}$. Then, solving eq. (39) and using eq. (41), we obtain

$$E(t) = E_0 \left(1 + \frac{1}{4}\frac{\sigma_{el} v_0}{\alpha_{in}} \ln(n_0/n)\right)^{-2}. \tag{42}$$

Here E_0 and $v_0 = (E_0/M)^{1/2}$ are the initial energy and velocity of $\bar{H}$, respectively.

Starting with $\bar{H}$ at 1 K this means that by the time the $\bar{H}$ density is reduced to 1% of its starting value as a result of rearrangement losses, the $\bar{H}$ can be cooled to 40 mK. Further cooling can be achieved only at the expense of even larger losses as the elastic scattering rate vanishes with decreasing temperatures as $\sqrt{T}$. Clearly the lifetime of the $\bar{H}$ is short in comparison to ordinary hydrogen. In spite of this somewhat disappointing situation, one should realize that, in real time, a proper choice of n_H should enable the observation of these processes *over the course of hours.*

Collisional cooling of $\bar{H}$ to 40 mK might be more simply arranged with $\bar{H}$–$\bar{p}$ elastic collisions by mixing $\bar{H}$ in a very cold $\bar{p}$ plasma. As for such collisions the condition $kR_e \approx 1$ corresponds to 40 mK the cooling occurs in the classical collision regime, where the elastic scattering leading to kinetic energy transport is determined by the capture in the effective potential $U_{\text{eff}}(R) = -\frac{1}{2}\alpha e^2/R^4 + E(\rho/R)^2$. The scattering rate constant $\alpha_c \approx 10^{-9}$ cm^3/s (see eq. (7)). Within a cooling model similar to that discussed above for the $\bar{H}$–H case, the cooling rate equation is

$$\dot{E} = -n_{\bar{p}}\alpha_c E/2, \tag{43}$$

where $n_{\bar{p}}$ is the $\bar{p}$ plasma density, and we obtain an exponential dependence

$$E(T) = E_0 \exp(-\tfrac{1}{2}n_{\bar{p}}\alpha_c t). \tag{44}$$

For $n_{\bar{p}} = 10^6$ cm^{-3} one calculates 5000 s to cool the $\bar{H}$ from 1 K to 40 mK.

4. $\bar{H}$ in the collisionless regime. The problem of one-dimensional adiabatic cooling

Let us now consider antihydrogen atoms in the collisionless regime and address the option of collisionless adiabatic cooling in which the characteristic energy of the antiatoms can be reduced by a slow continuous decrease of the potential energy in a magnetic trap. Such a system seems the most promising for achieving temperatures below 1 mK, since collisional cooling is not effective and laser cooling only works for $T > 1$ mK.

Decrease of the potential energy in the trap leads to expansion of the gas. For ordinary adiabatic cooling of a gas of colliding particles in thermal equilibrium, the characteristic energy per particle $E \sim V^{-2/3}$, where V is the characteristic volume of the system. From the experimental point of view it seems practical to increase the size of the system (decrease the potential energy) only in one dimension. In this case one should have $E \sim L^{-2/3}$, where L is the characteristic length of the dimension which is increasing. It is easy to estimate that such a situation is not very promising for cooling. For example, if we want to cool $\bar{H}$ from $T \sim 10$ mK (which can be reached by laser cooling) to $T \sim 100$ μK (relevant for gravity experiments) we need to increase L by three orders of magnitude. We emphasize that this conclusion does not hold for the collisionless regime unless there is sufficient coupling between the degrees of freedom. However, if the terms in the potential energy, mixing the motion along one axis of the coordinate system with the motion along the others, are small, the increasing size of only one dimension should, at least initially, lead to cooling in this particular dimension. As we shall show below, this situation can remain unchanged for large (and conceivably even infinite) times. Returning to our example we point out that in the case of one-dimensional collisionless adiabatic cooling the characteristic particle energy $E \sim L^{-2}$, and L has to be increased by only a factor 10 to reach 100 μK, starting at 10 mK.

We shall discuss the problem of one-dimensional cooling for the case of Ioffe magnetic trap proposed for neutral particles by Pritchard [18] and used in the hydrogen experiments [8–10]. The potential energy profile for $\bar{\mathrm{H}}\downarrow$ in the Ioffe trap is described by the relation

$$V = \{[\mu_B B_0 + \tfrac{1}{2}M_{\bar{\mathrm{H}}}\omega_z^2 z^2 - \tfrac{1}{4}M_{\bar{\mathrm{H}}}\omega_z^2(x^2+y^2)]^2 + \mu_B B_0 M_{\bar{\mathrm{H}}}\omega_r^2(x^2+y^2) + \tfrac{1}{4}M_{\bar{\mathrm{H}}}^2\omega_z^4 z^2(x^2+y^2) + (\mu_B B_0)^{1/2} M_{\bar{\mathrm{H}}}^{3/2}\omega_r\omega_z^2 z(x^2-y^2)\}^{1/2}, \tag{45}$$

where B_0 is the stationary external magnetic field, $M_{\bar{\mathrm{H}}}$ is the $\bar{\mathrm{H}}$ mass. We consider the case when the characteristic frequency of particle oscillations in the axial (vertical) direction is much smaller than the radial (horizontal) oscillation frequency,

$$\omega_z \ll \omega_r. \tag{46}$$

We confine ourselves to consideration of particle energies

$$E \ll \mu_B B_0, \tag{47}$$

which corresponds to $\mu_B B_0$ much larger than coordinate-dependent terms in eq. (45). Then, expanding (45) and retaining quadratic, cubic, and fourth-power terms in the coordinates, we arrive at the following expression for the trapping potential (the constant $\mu_B B_0$ is omitted):

$$V = \tfrac{1}{2}M_{\bar{\mathrm{H}}}\left(\omega_z^2 z^2 + (\omega_r^2 - \tfrac{1}{2}\omega_z^2)(x^2+y^2) + \omega_z\omega_r(x^2-y^2)\frac{z}{l_z} - \tfrac{1}{2}\omega_z^2 z^2(x^2+y^2)\left(\frac{1}{l_r^2} - \frac{1}{2l_z^2}\right) - \tfrac{1}{4}\omega_r^2(x^2+y^2)^2\left(\frac{1}{l_r^2} - \frac{1}{l_z^2}\right)\right). \tag{48}$$

Here

$$l_z = \left(\frac{\mu_B B_0}{M_{\bar{\mathrm{H}}}\omega_z^2}\right)^{1/2}, \tag{49}$$

$$l_r = \left(\frac{\mu_B B_0}{M_{\bar{\mathrm{H}}}\omega_r^2}\right)^{1/2} \ll l_z. \tag{50}$$

The inequality (47) leads to

$$z \ll l_z, \tag{51}$$

$$x, y \ll l_r. \tag{52}$$

Due to inequalities (51) and (52) the main coordinate-dependent terms in (48) are $\omega_z z^2$ and $(\omega_r^2 - \omega_z^2/2)(x^2+y^2)$ corresponding to the linear oscillator. Cubic and fourth-power nonlinear terms, in particular those which lead to mixing of radial and axial motion, are much smaller.

The form (48) for the trapping potential allows us to give a qualitative analysis of the problem of one-dimensional adiabatic cooling on the basis of general approaches of the theory of nonlinear dynamic systems. Let us consider one particle in the potential (48). The total Hamiltonian

$$H = \tfrac{1}{2} M_{\bar{\mathrm{H}}} (\dot{z}^2 + \dot{x}^2 + \dot{y}^2) + V \tag{53}$$

can be reduced to the normal Birkhoff form (see, e.g., ref. [19]), which allows us to establish adiabatic invariants of the motion by turning to angle-action variables. In our case it is possible to obtain the Birkhoff forms separately for the axial and radial motion, with the coupling terms in eq. (48) left out. The corresponding adiabatic invariants will be close to the linear oscillator ones, which for the axial motion is

$$J_z = \frac{E_z}{\omega_z}, \tag{54}$$

where E_z is the characteristic energy of the particle motion in the axial direction.

Considering axial motion with the coupling terms taken in account, we can treat these terms as perturbations proportional to a certain small parameter. Making a usual transformation (see ref. [19]) in V (48), for this parameter we obtain

$$\epsilon = \frac{\omega_z}{\omega_r} \left(\frac{E_r}{\mu_{\mathrm{B}} B_0} \right) \left(\frac{\mu_{\mathrm{B}} B_0}{E_z} \right)^{1/2} \tag{55}$$

(E_r is the characteristic energy of the radial motion). The parameter ϵ is really small for all energies E_z, $E_r \ll \mu_{\mathrm{B}} B_0$ except extremely small axial energies

$$E_z \lesssim E_r \left(\frac{E_r}{\mu_{\mathrm{B}} B_0} \right) \left(\frac{\omega_z}{\omega_r} \right)^2, \tag{56}$$

which correspond to characteristic values of the z-dependent quadratic term in eq. (48) smaller than those of the cubic one. These energies require a special consideration and are not discussed here.

In accordance with the general statement of the theory (see ref. [19]) J_z (strictly speaking, $\bar{J}_z$, corresponding to the axial motion with the pure z-nonlinearity taken in account) should remain approximately unchanged ($\Delta J_z / J_z \lesssim \epsilon$) at least on the time scale

$$\tau_z \sim (\omega_z \epsilon)^{-1}. \tag{57}$$

This statement also holds if ω_z slowly changes (on the time scale $\tau \gg \omega_z^{-1}$). Then J_z should approximately remain unchanged on the time scale $\min(\tau_z, \tau)$. So, the energy of the axial motion, E_z, is approximately proportional to the axial frequency ω_z and continuously decreases with decreasing ω_z. In this process the time τ_z increases proportional to $\omega_z^{-3/2}$ as follows from substitution of eqs. (54) and (55) into eq. (57).

Our qualitative conclusion is confirmed by computer calculation with the trapping potential in the original form (45). Moreover, the calculation shows that J_z does not change appreciably on a time scale by an order of magnitude larger than τ_z. The existence of approximate adiabatic invariants at much larger and infinite times still remains questionable and will be a subject of our future investigations.

However, already for the parameters of the Ioffe trap currently used in hydrogen experiments [20] ($\omega_z \sim 10^3\,\mathrm{s}^{-1}$, $\omega_r \sim 3 \times 10^4\,\mathrm{s}^{-1}$, $\mu_B B_0 \sim 0.1$ K) the time scale $\sim 10\tau_z$ is of order 1 s for $E_r = E_z = 10$ mK and should increase to ~ 1000 s in the process of adiabatic cooling to $E_z = 100\,\mu$K by slowly decreasing ω_z to 1% of the quoted value. This shows a real possibility to observe one-dimensional cooling experimentally and to use this phenomena for gravity experiments with $\bar{\mathrm{H}}$.

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