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BEC and the Relaxation Explosion in Magnetically Trapped Atomic Hydrogen

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Abstract

We predict and analyze non-trivial relaxational behavior of magnetically trapped gases near the Bose condensation temperature T_c. Due to strong compression of the condensate by the inhomogeneous trapping field, particularly at low densities, the relaxation rate shows a strong, almost jump wise, increase below T_c. As a consequence the maximum fraction of condensate particles is limited to a few percent. This phenomenon can be called a "relaxation explosion". We discuss its implications for the detectability of BEC in atomic hydrogen.

Magnetostatic traps offer the possibility to study gases of Bose particles in the truly dilute limit, and have proved particularly fruitful [1, 2, 3, 4, 5] in the study of atomic hydrogen (H). In these traps, proposed for H by Hess [6], the effective elimination of physical boundaries is accomplished by creating a magnetic field minimum in free space. This minimum forms a potential well for electron spin-up polarized atoms (H↑), called low-field seekers. The occurrence of Bose–Einstein condensation (BEC) in such systems introduces qualitatively different behavior from the case of a homogeneous Bose gas. This is related to the explosive increase of the dipolar relaxation rate associated with the strong compression of the condensate in an external potential. A similar phenomenon occurs in connection with three-body recombination in high density systems [7] The large condensate density resulting from this compression gives rise to an increase of the rate of inelastic (dipolar) pair collisions, in which spins of the colliding particles are flipped, producing high-field seeking

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atoms (H↓) which are ejected from the trap. The approach to BEC is often [6, 8] described in terms of trajectories in density–temperature space. We shall see that in H, the increase in relaxation rate resulting from the appearance of the condensate, the "relaxation explosion", prevents one from penetrating deep into the BEC region and markedly alters these trajectories.

For simplicity we consider an isotropic harmonic trapping potential of the form

$$V(r) = \mu_B B_0 + \frac{m}{2}\omega^2 r^2, \tag{1}$$

where r is the distance to the trap center, and ω is the oscillation frequency. The critical BEC temperature is expressed by the relation $T_c = 3.31\hbar^2 n^{2/3}/m$, where n is the density of the gas at the trap center and m is the mass of the atom. The density profiles in external potentials were analyzed by Goldman *et al.* [9], and by Huse and Siggia [10]. The critical temperature can be expressed in terms of the total number of particles N, and the parameters of the trapping potential [11]. For the potential given by Eq. (1) we have

$$T_c = (N/g_3(1))^{1/3}\hbar\omega. \tag{2}$$

Here $g_3(1) = 1.20$, where g_ℓ is a Bose integral given by $g_\ell(\xi) = \sum_{n=1}^{\infty}(\xi^n/n^\ell)$.

Well above T_c, the number of relaxation events per unit time is given by

$$\nu_r = 2\alpha \int \mathrm{d}\mathbf{r}\, n^2(r), \tag{3}$$

where $n(r)$ is the density distribution and α is the rate constant for dipolar decay. For hydrogen $\alpha \approx 10^{-15}$ cm^3/s [12, 13]. Below T_c we should replace Eq. (3) by a more general expression [14, 15]:

$$\nu_r = 2\alpha \int \mathrm{d}\mathbf{r}\, \frac{1}{2} < \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}) >, \tag{4}$$

where $\hat{\psi}$ is the field operator of the atoms and $< \hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi} >$ is the local two-particle correlator.

A detailed calculation [16] shows that, below T_c, the relaxation rate can be expressed in terms of its value at T_c:

$$\nu_r(T) = \nu_r(T_c)(1 + 7.5(\frac{T_c}{n_c\tilde{U}})^{3/5}(\frac{\Delta T}{T_c})^{7/5}) \;\; ; T \leq T_c \tag{5}$$

Here, $\Delta T = T_c - T$, and $n_c\tilde{U}$ represents the mean field interaction

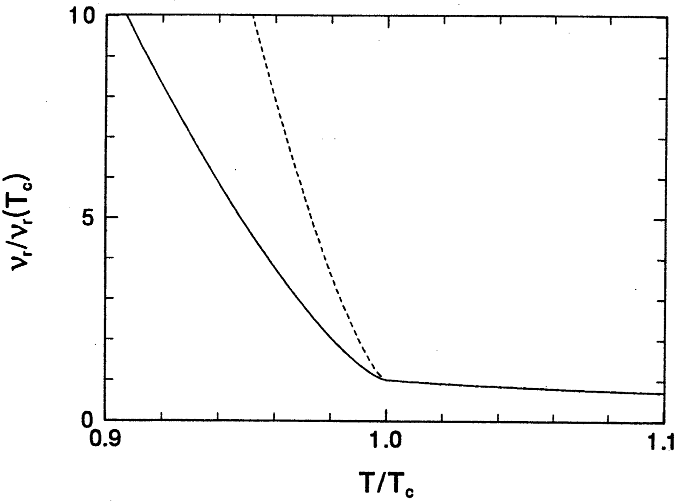

Fig. 1. Number of relaxation events per unit time for a fixed number of atoms, normalized to the value at T_c. The solid line corresponds to $n_c = 10^{15}$ cm^{-3} and the dashed line to $n_c = 10^{13}$ cm^{-3}.

energy, where n_c is the critical density. For H, at $T = 100$ μK, we have $n_c \approx 5 \times 10^{14}$ cm^{-3} and $n_c\tilde{U} \approx 0.2$ μK. This clearly shows the extreme weakness of the interaction in this system.

Qualitatively, Eq. (5) can be understood as follows. Above T_c we have, according to Eq. (3) $\nu_r \sim \alpha N^2/V_e$, where the classical effective volume V_e is defined trough $n(r) = (N/V_e)\exp(-V(r)/T)$. Below T_c we have an additional contribution associated with the condensate particles: $\nu_0 \sim \alpha N_0^2/V_0$. Explicit calculation [16] shows

$$V_0 = \frac{4}{3\sqrt{(\pi)}}(\frac{\mu - 2n_c\tilde{U}}{T_c})^{3/2}V_e \approx 2.0(\frac{n_c\tilde{U}}{T_c})^{3/5}(\frac{\Delta T}{T_c})^{3/5}V_e. \qquad (6)$$

Clearly, as a result of the weak interaction, we have $V_0 \ll V_e$. Hence, even if $N_0 \ll N$ (and accordingly $\Delta T \ll T$) the condensate can dominate the relaxation rate. In Fig. 1 the value of $\nu_r(T)/\nu_r(T_c)$ is given for H atoms for $n_c = 10^{13}$cm^{-3} ($T_c = 7\mu$K) and $n_c = 10^{15}$cm^{-3} ($T_c = 150\mu$K).

To investigate the effect of the relaxation explosion, we use a simple model for the kinetics of the cooling proces, in which we assume that the cooling proceeds sufficiently slowly to consider the system as being in thermal equilibrium. To obtain the trajectories in $N - T$ space, we start from the expressions for the internal energy of the trapped gas at temperatures above and below T_c (here we neglect $n_c\tilde{U}$):

$$U = 3NT\frac{g_4(\xi)}{g_3(\xi)} \quad ; T \geq T_c \tag{7}$$

$$U = 3NT(\frac{T}{T_c})^3\frac{g_4(1)}{g_3(1)} = 3g_4(1)\frac{T^4}{(\hbar\omega)^3} \quad ; T \leq T_c, \tag{8}$$

where ξ is the fugacity $\exp(-\mu/T)$. From this we obtain relations between the time derivatives of T, U and N:

$$\left(4\frac{g_4(\xi)g_2(\xi)}{g_3^2(\xi)} - 3\right)\frac{\dot{T}}{T} = \frac{\dot{U}}{U}\frac{g_2(\xi)g_4(\xi)}{g_3^2(\xi)} - \frac{\dot{N}}{N} \quad ; T \geq T_c. \tag{9}$$

$$\frac{\dot{T}}{T} = \frac{1}{4}\frac{\dot{U}}{U} \quad ; T \leq T_c. \tag{10}$$

If we parametrize the cooling process by an energy removal rate $R_U \equiv -\dot{U}/U$, and an accompanying rate R_N of particle removal, we can write the rate of change of particle number and internal energy as

$$\frac{\dot{N}}{N} = -R_N - \frac{\nu_r}{N}; \quad \frac{\dot{U}}{U} = -R_U + \frac{\dot{U}_r}{U}. \tag{11}$$

Here $\dot{U}_r$ is time rate of change of energy associated with relaxation. As the relaxing particles have a finite energy, the internal energy decreases with decreasing particle number. Hence, $\dot{U}_r$ is always negative. From Eq. (10) one thus finds that when $T \leq T_c$, because the energy does not explicitly depend on N, relaxation does not lead to heating. In contrast, from Eq. (9) we see that for $T > T_c$ removal of particles only leads to cooling if the energy of these particles is sufficiently high. This is the principle of evaporative cooling. In this sense, any loss of (above-condensate) particles leads to evaporative cooling for $T \leq T_c$. Note, moreover, that the ratio $\dot{T}/\dot{N}$ changes discontinuously at T_c. This behavior has the same origin as the discontinuity in the derivative of specific heat at the BEC point. A straightforward calculation shows that $\dot{U}(T) = -(9/4)T\nu_r$ for $T \gg T_c$ and $\dot{U}(T) = -(9/4)T0.8\nu_r(T_c)$ for $T < T_c$.

Using the above results we can easily obtain the trajectories in the $N - T$ plane, if we make the somewhat simplifying assumption that R_U is proportional to the elastic collision rate $R_c \equiv \frac{1}{4}\sigma v_T N/V_e$. The resulting trajectories are shown in Fig. 2 for two values of $\tilde{R}_U \equiv R_U/R_c$. Below T_c the relaxation explosion manifests itself as an attraction of the trajectories towards the BEC line. This is particularly striking when comparing with the corresponding trajectories obtained by assuming the system to obey Boltzmann statistics at any temperature and density.

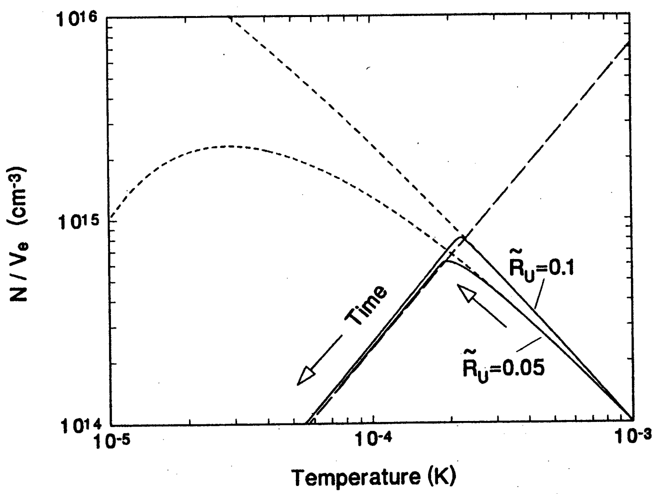

Fig. 2. Cooling trajectories for $\tilde{R}_U = 0.1$ and 0.05 (solid lines) plotted as N/V_e vs T. The long-dashed curve is the BEC line. The short-dashed curves represent the trajectories corresponding to a system obeying Boltzmann statistics.

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